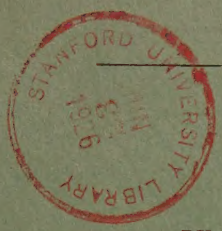
# AN APPROXIMATE DETERMINATION OF THE DEPTH OF COMPRESSION IN COPPER BARS DURING IMPACT




BY

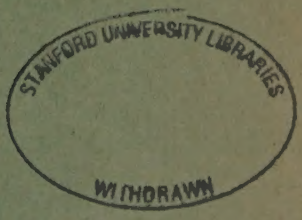
E. W. TSCHUDI

DISSERTATION

SUBMITTED TO THE BOARD OF UNIVERSITY STUDIES OF THE JOHNS HOPKINS UNIVERSITY IN CONFORMITY WITH THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

June

1925.




3781
.65T
CLASSED
Dissertation

# AN APPROXIMATE DETERMINATION OF THE DEPTH OF COMPRESSION IN COPPER BARS DURING IMPACT

---

BY

E. W. TSCHUDI

---

DISSERTATION

SUBMITTED TO THE BOARD OF UNIVERSITY STUDIES OF THE JOHNS HOPKINS UNIVERSITY IN CONFORMITY WITH THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

---

June

1925.

# AN APPROXIMATE DETERMINATION OF THE DEPTH OF COMPRESSION IN COPPER BARS DURING IMPACT

By E. W. Tschudi

Abstract

**Duration of impact of copper bars and of copper plated steel bars.**— As in previous experiments, the duration of impact was measured by means of a Duddell-type oscillograph connected in series with the pair of colliding bars, each of which was suspended by four wires adjusted so that the plane ends struck squarely. In all cases the mass of each bar tested was the same, 2400 gm. By trying three diameters of contact area $d$, it was found that the time varies as $1/d^{1/2}$; it also varies as $1/v^{1/5}$; hence $t=A/d^{1/2}v^{1/5}$, where $A$ has the values $9.7\times10^{-4}$ and $12.35\times10^{-4}$ c.g.s. units for these steel and copper bars. The steel bars were then plated with coherent copper deposited electrolytically to thicknesses of .017, .038, .064 mm. The time for the two thick films came out the same as for solid copper, while that for the thinnest film was between that for copper and that for steel. These results indicate that the thickness of the surface layer whose elastic constants determine the time of impact is only about .03 mm. In the case of spherical ends of 1.27 cm radius, impact at a velocity as low as 10 cm/sec. resulted in flattening the ends, the pressure exceeding the elastic limit for steel.

## Introduction

TWO distinct experiments have been previously reported which led to the conclusion that the separation of two bars, with plane ends, after an end-on collision was due to a local compression in the immediate vicinity of the common surface of contact and not to a compressional wave which traversed the length of each bar, was reflected and returned to the contact ends. The first[1] of these was performed with a slow speed model of colliding bars which consisted of two helical springs with each coil weighted and the colliding ends fitted with brass annular rings. It was found that the springs separated when the compressional wave had traversed but one fourth the length of each spring.

In the second experiment[2] a steel bar was made to strike against another cylindrical piece of steel whose mass equalled the mass of the bar but whose length was only one sixth that of the bar. The duration-velocity curve for this combination was identical with the curve for two similar bars. In general, the nature of all duration-velocity curves indicated that the duration of impact bore a definite relation to the initial velocity of impact and was not a constant for all velocities as predicted by the com-

[1] Tschudi, Phys. Rev. **18**, 423 (December 1921).
[2] Tschudi, Phys. Rev. **23**, 756 (June 1924).

pressional wave theory. Wagstaff[3] aimed to show, by the use of bars with rounded ends, that both the compressional wave and local compression enter into the problem, the former predominating in the case of very long bars and the latter alone controlling the process in the case of short bars, i.e., that the shorter the bars the more nearly does Hertz's theory[4] of local compression for colliding spheres hold. The author has found, as will be shown in this paper, that the use of bars with rounded ends proved unsatisfactory because of stresses which exceeded the elastic limit; and further, that for plane ends the duration of impact varies with the area of contact at any given velocity.

These previous experiments suggested the problem of determining the order of the depth of compression. This was the purpose of the present experiments which were performed with copper bars and with steel bars of the same mass with their colliding ends copper plated.

## The Apparatus

The bars were each suspended in much the same manner as in the earlier work,[2] from knife edge supports by two bifilar suspensions. The upper end of each strand of these suspensions terminated at an adjusting screw which permitted a four fold adjustment for each bar. An electromagnetic release held one of the bars, drawn aside, in the proper position from which it could be released at will. The length of time in which an electric current flowed through the bars during a collision determined the duration of the impact. It was found that additional wires, used for current carriers, hindered the motion of the bars and altered the value for the duration of impact. Hence the suspending strands themselves were used to carry the current. The relative initial velocity of impact was calculated from the height through which the deflected bar dropped. (The second bar was kept at rest initially.) This height was determined with a cathatometer which measured accurately to thousands of a centimeter.

In the present experiments an oscillograph was employed for measuring the duration of impact. A special camera which required a five-foot length of film was employed. The peripheral speed of the camera drum was approximately 10 meters per second. No effort was made to damp the element critically, as was done in the earlier experiments because the contact resistance was of no concern here. Since the camera shutter was open for only one revolution of the drum, close coordination was required between the release of the deflected bar and the opening of the shutter.

[3] Wagstaff, Proc. Roy. Soc. A105, 544 (1924); also Phil. Mag. **48**, 147 (1924).
[4] Hertz, Miscellaneous Papers, p. 146, 1896.

This was accomplished by the use of a single-pole double-throw switch which in the maximum displacement of the pole was about 2 cm in the vertical. With the pole in the neutral position, the one bar drawn aside, and the camera drum rotating, a catch was pushed aside which allowed the pole to fly upward under the influence of a spring. The circuit containing the electromagnet was thereby closed and the bar released. An instant before the two bars came in contact, the pole was pushed down through the neutral position and the circuit which actuated the camera shutter was closed.

The circuit containing the two bars and oscillograph element contained no inductance, but a resistance of four lamps in parallel was required to protect the element. A two volt cell supplied the requisite current.

## The Experiments

Before any records could be obtained, it was of the utmost importance to adjust the bars carefully. Not only was it necessary that the two bars be horizontal but, far more important, that their colliding ends be parallel. These conditions were obtained by the use of the eight adjusting screws. Adjustments were completed only when no light could be seen through the junction of the colliding ends, with both bars in their equilibrium positions, when observed in two directions perpendicular to each other. The stand of the electromagnetic release was first stationed so as to permit the bar which was to be deflected, to be raised to the desired height and then shifted laterally until its stop and the opposite ends of the bars fell in the same line. This latter adjustment, if properly made, assured a collision in which the colliding ends hit true. When this was not accomplished the oscillogram registered this fact, and a small shift of the stand corrected the fault.

Two satisfactory exposures were obtained for each velocity of impact before proceeding with another velocity. A duration-velocity curve was obtained for a pair of copper bars of length 54.2 cm, diameter 2.54 cm, and mass 2430 gm. A similar curve was obtained for a pair of steel bars of the same diameter and practically the same mass, but of length 61.3 cm. These curves are both given in Fig. 1.

The steel bars were then fitted with copper plugs so that the bars appeared to have a thickness of 0.32 cm of copper on the colliding end of each. The purpose of these plugs was to determine if that thickness was sufficient to yield the same results as those of copper bars. Unfortunately, it was difficult to screw these plugs into the steel bars sufficiently tightly without marring the faces of the plugs. Nevertheless, a series of tests was

made which resulted in a smooth curve that lay far above the copper curve at very small velocities but came close to it at larger velocities.

Since copper cannot be deposited upon steel from a cupric solution it was necessary to deposit a copper base from a cuprous solution. The following recipe was used in preparing the cuprous solution: water, 3.75 liters; sodium cyanide, 64.0 gm; copper cyanide 56.8 gm; ammonium chloride 9.5 gm. Before employing this solution, the customary care was taken to clean thoroughly the ends of the steel bars that were to be plated. The cleaning process consisted in rubbing the surfaces with fine emery paper then dipping them into a boiling solution of concentrated potassium hydroxide, rubbing with scouring powder, dipping again in the alkaline solution and finally rinsing in tap water.

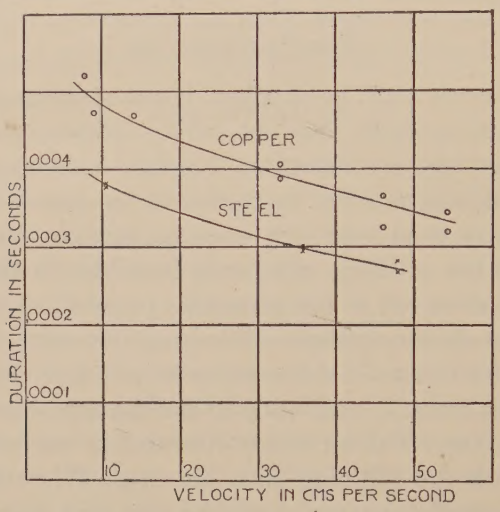



Fig. 1. Duration of impact as a function of velocity of impact, for steel and copper bars.

In the electrolytic bath, the copper anode was flat on the bottom of the vessel. The bar was thrust into the solution sufficiently to wet only the surface to be plated and was clamped in a vertical position. The surface tension of the liquid caused it to climb up the side of the bar about 0.32 cm. A current of 0.1 ampere was sent through the cell for thirty minutes. This length of time was found to be a minimum in order to deposit a sufficient amount of copper to form a base for the deposition from a cupric solution. It was impossible to use the cuprous solution to deposit the requisite thickness of copper because this deposition was found to be of a spongy nature when continued over a period of several hours. The deposition from the cupric solution upon the base from the cuprous solution was, however, of the same smooth, lustrous character as when deposited upon a pure piece of copper.

After removal from the cuprous cell, the plated portion of the bar was cleaned as before with the exception of the rubbing with emery paper. In the cupric cell, a current of .05 ampere was maintained. A copper sulphate solution was used in this cell.

The duration-velocity curve for the bars with the first deposition from the cupric solution is given in Fig. 2, where also the curve for the copper bars is repeated. We have here a comparison of pure copper bars with steel bars with copper plated ends, each having the same mass and diameter, so that the impact relations are practically the same.

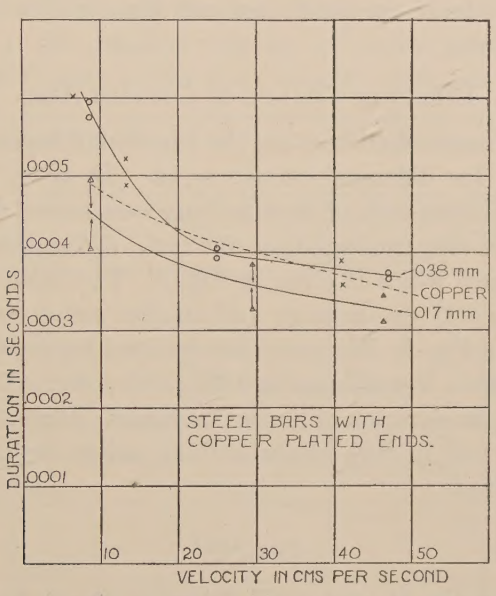



Fig. 2. Effect of plating steel bars with copper, on the duration of impact.

An additional layer of copper was then deposited from the cupric solution, with the purpose of determining whether a thicker layer would produce a duration-velocity curve which lay above the one for the first layer. This, however, was not found to be the case. The points corresponding to the thicker layer are indicated by small crosses in Fig. 2. The closeness of these points to the first curve indicates that the first thickness of copper was sufficient to produce results similar to those of pure copper bars.

The copper deposition was next removed from the steel bars in a lathe. It is worthy to note that the plated copper was found to be firmly fastened to the steel and did not peel off under the tool.

Again the ends of the steel bars were plated from the cuprous solution, but no additional copper was deposited from the cupric solution. Another

series of tests were made. The resultant duration-velocity curve, given in Fig. 2, fell below the curve for pure copper bars but above the curve for steel bars with unplated ends.

In calculating the thickness of copper deposited it was assumed that the form of the deposition was in the shape of a cup 0.32 cm deep with walls and bottom of the same thickness. The first deposit of copper was then computed to have a thickness of .038 mm, the second .064 mm, and the third .017 mm. One can conclude then that the equivalent thickness of deposited copper on steel bars which will give the same impact results as pure copper bars of the same mass and diameter lies between .02 mm and .04 mm.

## Effect of Change of Contact Area

The question arose whether or not the duration of impact varied, at any given velocity, for different contact areas. In order to answer this question, the colliding ends of the steel bars were turned down to 1.90 cm for a distance of 0.32 cm back from the end. A duration-velocity curve was then obtained which is given in Fig. 3. The same ends were then turned down to 1.27 cm diameter and another curve obtained which is likewise given in Fig. 3. The curve for the steel bars with diameter 2.54 cm is repeated here. It will be noticed that with a decrease in the common area of contact the duration of impact increases. Evidently the duration of impact of a pair of bars means nothing unless the common area of contact is specified.

The relation

$$t = c_1/d^{1/2} \tag{1}$$

where $t$ is the duration of impact and $d$ the diameter of the circular contact area, agrees with experimental curves very well. In the following tables it will be seen that, for any definite velocity, $c_1$ is nearly constant.

| $v=10$ | | $v=20$ | | $v=30$ | |
|---|---|---|---|---|---|
| $d$ | $c_1\times10^4$ | $d$ | $c_1\times10^4$ | $d$ | $c_1\times10^4$ |
| 2.54 cm | 6.12 | 2.54 cm | 4.57 | 2.54 cm | 4.94 |
| 1.90 | 5.85 | 1.90 | 4.80 | 1.90 | 4.96 |
| 1.27 | 6.19 | 1.27 | 5.32 | 1.27 | 5.02 |

Likewise, for any given contact area, the relation

$$t = c_2/v^{1/5} \tag{2}$$

holds admirably for all cases. This form of the velocity function has been agreed upon by all investigators, whether they used spheres, bars with rounded ends, or bars with plane ends. Combining (1) and (2) we have

$$t = A/d^{1/2}v^{1/5} \tag{3}$$

The coefficient $A$ is not a universal constant, but varies with the substance. Its values for steel and copper are $9.7 \times 10^{-4}$ and $12.35 \times 10^{-4}$ respectively, in c.g.s. units.

The steel bars were next rounded so as to have a hemispherical surface of 1.27 cm radius. Upon allowing the bars to collide, it was noticed that the clear metallic ring which had accompanied all previous collisions with plane ends, was lacking, and instead that a dull thud sounded as though two bars of lead had collided. Examination of the rounded ends showed a small flat area of about 1/2 mm diameter, indicating that the elastic limit had been exceeded and the ends of the bars had been mashed in. This occurred at the smallest velocity at which measurements were made. Successive blows at the same velocity increased the area of the

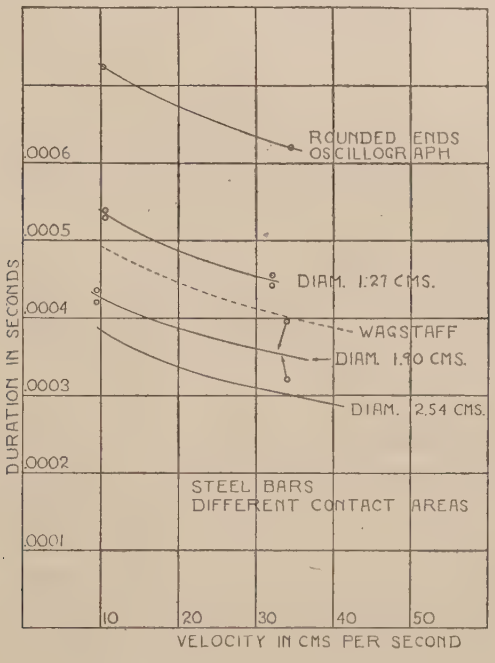



Fig. 3. Effect of contact area on duration of impact.

flattened portion. Examination of the oscillograms showed that there was no definite point at which the current had broken and therefore no measurement of the duration of impact could be made. A condenser method for measuring the duration of impact would not have disclosed the fact that the impact had been erratic, which emphasizes the necessity of an oscillograph for making measurements of this kind.

The bars were again rounded so as to expose unstrained surfaces and allowed to collide at the same velocity as before. Three successive

collisions were recorded. The resultant oscillograms proved satisfactory this time. The duration of impact, as measured from them, was found to grow smaller as the number of impacts increased, due, no doubt, to the increase in contact area. Similar results were obtained at a greater velocity. They are tabulated below.

| $v=10.28$ | | $v=34.80$ | |
|---|---|---|---|
| Trial | Duration | Trial | Duration |
| 1 | $7.23\times10^{-4}$sec. | 1 | $6.20\times10^{-4}$sec. |
| 2 | 6.82 | 2 | 5.32 |
| 3 | 5.75 | 3 | 4.35 |

The durations of impact for the first trial at each of the two velocities are plotted in Fig. 3. In the same figure is plotted a broken curve for steel bars, of practically the same dimensions, from values given by Wagstaff,[3] who used a condenser discharge method for measuring the duration of impacts.

In the classical relation between the impulse and change in momentum of colliding bodies, if it be assumed that the force during the impact varies as $F=B \sin (\pi t/t_1)$ where $t_1$ is the duration of the impact, then it can readily be shown that the maximum force exerted during impact is $B=(mv-mu)\pi/2t_1$. Substituting values from the preceding table, for example, $t_1=7.23\times10^{-4}$, $v=10.28$, we have $B=5.22\times10^{7}$ dynes. This force is distributed over an area of $1.96\times10^{-3}$ cm.$^2$ Hence the force per unit area is computed to be $2.66\times10^{10}$ dynes, more than twice the maximum limit of elasticity tabulated for steel.

The author is greatly indebted to the Bureau of Standards for the loan of the oscillograph used in these experiments and takes this opportunity to express his thanks and obligation. Suggestions and criticisms offered by Dr. J. S. Ames during the course of the experiments were of high value and greatly appreciated.

Johns Hopkins University
March 18, 1925.

## BIOGRAPHICAL NOTE

Erwin William Tschudi, son of Henry and Minnie (Chiseck) Tschudi, was born June 2, 1898, at Cincinnati, Ohio. His early education was acquired in the North Fairmount Public School, and his secondary education in Hughes and Woodward High Schools. During summer months, until the beginning of his junior year at College, he spent his time learning the machinist's trade under the supervision of his father, who was in charge of a small tool factory in the town of Norwood, Ohio.

He entered the Liberal Arts College of the University of Cincinnati in September, 1915, and received the A.B. degree in June, 1920. During the summer following his sophomore year he served as an instructor in Machine Shop Practice in a temporary army school. During the first three months of his junior year he was a member of the S.A.T.C. of the U.S.A. Throughout the junior year he acted as reader in Mathematics. As a senior he taught Mathematics in the Engineering College and assisted in the Physics Laboratory.

During 1920-21 he was a graduate student in Physics and Mathematics, and at the same time taught Mathematics. He received the A.M. degree in June, 1921, from the University of Cincinnati. The next year was spent as a graduate student and instructor in Mathematics at the University of Illinois.

In September, 1922, he entered The Johns Hopkins University as a graduate student in Physics where he studied for three years under Professor Joseph S. Ames, at the same time acting in the capacity of Instructor in Physics.

During the summer of 1923 he was employed as Assistant Physicist at the Bureau of Standards, and during the summer of 1924 as Physicist at the Bureau of Entomology in Washington, D.C.